A Preliminary Study of 2D Leaflet Motions in the Left Ventricle using Arbitrary Lagrangian – Eulerian (ALE) Approach

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Current flow pattern simulation in the left ventricle requires method to tackle the interaction between the moving blood flow and the deforming valves structures. Capturing essence of these mechanics are feasible with the use of mathematical formulations derived from the differential domains. The degree of complexity stemming from these equations often requires high computing resources. For this reason, the equations are first modeled on two-dimensional (2D) geometries to optimize the degree of freedom and limit the computing power. Enhancement of the prototype into 3D is usually done later to produce better graphical presentation. In this research, the dynamic movement of the heart valves and the blood flow pattern in the left ventricle is studied using advanced numerical-computational technique known as the Arbitrary Lagrangian-Eulerian (ALE). The ALE uses two sets of reference frames; the Eulerian reference frame to tracks the flowing blood and the Lagrangian reference frame to handle the deforming leaflets. An arbitrary grid is used as base for the reference frames to compute simulations of the fluid-structure interaction. A working prototype of the ALE model depicting the leaflet motions in 2D is developed and results pertaining to the velocity pattern prior to periodicity mechanics of the leaflets are presented in a time-step fashion.

1. Introduction

The heart functions as the organ that generates force which circulates blood in the body. The force is generated from the dynamic interaction between the fast flowing blood and the elastic heart wall in the left ventricle. In each heart beat, the blood would fill the left ventricle and the elastic left ventricle wall would strain and expand and the building constraint would generate high pressure enough to force the blood out from the heart and into the body. The journey of the blood into the left ventricle and out into the body creates a profile of flow pattern as seen in Fig. 1.

Information on the general pattern of the flowing blood in the left ventricle is currently retrieved using the cardiac catheterization; a technique performed on potential patients requiring a small incision and sedation. Unfortunately, evaluation of patients with a potential circulatory disorder using this technique is often very difficult, long (the whole process takes up several hours) and invasive to make. Therefore, alternative methods using computational techniques capable of determining the flow pattern in the heart, non-invasively, have triggered a strong interest to clinical and physiological diagnosis.

1.1 Literature Review

In modern studies of the fluid-structure interactions of the left ventricle, several researchers around the world have been identified to incorporate the ALE approach in their work.

- A group of researchers from the Eindhoven University of Technology has developed tissue engineered heart valves by analyzing how the geometry and material properties influence the movement of artery and heart valves and the flow vicinity in the heart. (Ref. 1)

- Simulations of valve dynamics in a simplified left ventricle flow model using the ALE method can be seen in. (Ref. 2)

- The ALE approach has also been used to study the fluid-structure analysis of the structural and the fluid dynamic behavior for cardiopulmonary bypass. (Ref. 3)

- Another work incorporating the Lagrangian-Eulerian formulation to study the incompressible viscous flow on a finite element platform can be seen in. (Ref. 4)

- Fluid-solid interaction of the valves leaflets motions has been investigated by means of the ALE couplings to derive two-dimensional model of the bi-leaflets valve (Ref. 5) and three-dimensional model of the bi-leaflets model (Ref. 6)

- Numerical analysis of fluid-solid interactions for blood flow on arterial structures is paid careful attention in some works (Ref. 7) and (Ref. 8). From these experimental works, additional information pertaining to the heart’s boundary positions and velocity information can be observed.

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Fig. 1: Profile of flow pattern in the left ventricle blood circulation (Price and Wilson 1992)
2. Methodology

2.1 Geometrical Modelling

A simplified surface model of the heart is needed for the purpose of preparation and acceptance of the 2D ALE analysis and modelling. Using 3D scanning, points data that represents the left ventricle is captured and used to recreate a sensible geometrical model. Expectations of the outcome from this session are for the model to encompass parameters that contributes to proper measurements of the fundamental aspect of the left ventricle such as the general morphology of its walls in axial direction, diameter and shape of the ventricle valves (i.e. the mitral and the aortic), and the radius angle of those working valves that positions them at such in the chamber.

Framework used in geometrical modelling is summarized as follows:

1. Sample left ventricle points data from 3D scanning

   Fig. 2.1: Generation of points cloud data of the inner layer of the left ventricle with 3375 points collected.

2. Perform noise reduction to retrieve approximate data

   Fig. 2.2: Results from curvature sampling on the surface model at (from left) 30% curvature sample, 65% curvature sample and 100% curvature sample.

3. Wrap points data using computer aided design (CAD) software.

   Fig. 2.3: Results from polygonal surface reconstruction (from left) at 10mm, 5mm and 2mm point distance.

4. Generate surface on the 3D geometry.

   Fig. 2.4: Results from surface wrapping on the left ventricle data. Surface wrapping at 10mm, 5mm and 2mm point distance.

5. Measure length and diameter of regions of interest.

   Fig. 2.5: Regions of interest considered for the left ventricle measurements. On the left; length and diameter measurements and on the right; the crease angle calculation

6. Design 2D geometrical model based on 3D surface model measurements.

   Fig. 2.6: 2D solid boundary model of the left ventricle.

2.2 Mathematical Modelling Concept

The generated model is later tuned in a format that can be recognized in an arbitrary environment. Here, governing equations controlling the movement of the fluid flow and structure deformation are defined on the left ventricle geometry model with careful attention. The arbitrary grid uses the Eulerian reference frames to accept the blood or fluid properties such the viscosity, initial velocity, and density of the blood (see Table 1) and the Lagrangian reference frames is assigned with structural information for instance the displacement and elastic properties of the leaflet boundaries (see Table 2). Finite element setting has been suggested as suitable computational platform for the arbitrary grid which recognizes both reference frames in the ALE simulation.

2.3 ALE Analysis

The ALE approach is adopted in this work to solve two problems; the fluid problem (i.e. the motion of the blood) and the structure problem (i.e. the movement of the valves). The method employs the use of reference frames to represent the Lagrangian and Eulerian systems. The Lagrangian reference frame is used to study the structure problem and the Eulerian reference is used to study the fluid problem, respectively.

2.3.1 The Eulerian Reference Frame

The Eulerian reference frame studies the motion of fluid by setting its frame on the fluid mesh and allows the fluid to cross through the mesh to study the flow. Fluid has velocity and so, as the fluid flows through the mesh, the Eulerian calculation measures the differing velocity in and out of each frame section. This measurement captures velocity information of the blood at various locations in the left ventricle geometry. Important parameters pertaining to the flow is summarized in Table 1.
### Table 1 Field Variables Representing the Mesh Velocity

<table>
<thead>
<tr>
<th>Mesh Velocity Field Variables</th>
<th>Values</th>
</tr>
</thead>
<tbody>
<tr>
<td>Density (ρ)</td>
<td>1060 Kgm/m³</td>
</tr>
<tr>
<td>Dynamic Viscosity (ν)</td>
<td>0.0054 Pa-1s</td>
</tr>
</tbody>
</table>

PDE of the mesh velocity is derived based on the Navier-Stokes equations and has been evaluated numerically with respect to time. The equation is presented as follows:

$$\rho \frac{\partial \mathbf{v}}{\partial t} + \rho \mathbf{v} \cdot \nabla \mathbf{v} = -\nabla p + \mu \left( \nabla \mathbf{v} + (\nabla \mathbf{v})^T \right) + f + \mathbf{g}$$

(1)

where $\mathbf{v} = (\psi_x, \psi_y)$ is the mesh velocity.

### 2.3.4 The Lagrangian Reference Frame

The Lagrangian reference frame investigates structural deformation by fixing its frame on the mesh structure then as the structure deforms the mesh deforms with it. The deformation captured by the mesh is the structural displacement of the valve leaflets. Important parameters pertaining to the mesh displacement is summarized in Table 2 below.

### Table 2 Field Variables Representing Properties of the Mesh Displacement

<table>
<thead>
<tr>
<th>Mesh Displacement Field Variables</th>
<th>Values</th>
</tr>
</thead>
<tbody>
<tr>
<td>Density (ρ)</td>
<td>960 Kgm/m³</td>
</tr>
<tr>
<td>Coefficient (u)</td>
<td>6204106 N/m²</td>
</tr>
<tr>
<td>Bulk Modulus</td>
<td>20u</td>
</tr>
<tr>
<td>Poisson Ratio (ν)</td>
<td>0.45</td>
</tr>
<tr>
<td>Elastic Modulus</td>
<td>1.017 N/m²</td>
</tr>
</tbody>
</table>

PDE for the mesh displacement is derived based on the information captured from Eq. (1). Poisson’s equation is used to represent the displacement equation.

$$\int_{\Omega} D(\psi_x \frac{\partial I_{x\psi}}{\partial X_x} + \psi_y \frac{\partial I_{y\psi}}{\partial X_y} + \psi_x \frac{\partial I_{x\psi}}{\partial X_y} + \psi_y \frac{\partial I_{y\psi}}{\partial X_x}) d\Omega = 0$$

(2)

### 2.3.5 The Arbitrary Grid

In view of the fact that the mechanics in the heart requires simultaneous interaction between the valves movement and the blood flow, solution to the interaction is derived by setting the Lagrangian and Eulerian reference frames on an arbitrary computational grid. Coupling of both reference frames on the grid produces solution to the fluid-structure problem and a graphical simulation can be derived using visualization platforms to output the interaction. Summary of the ALE algorithm is shown in Fig. 2.7.

### 3. Discussion

The influence of the modulus elasticity particularly on the walls or boundaries of the working valves has posed attention to the analyzed result relating to the calculation of the Reynolds numbers; a dimensionless parameter identified from the simulation with a value of 500, which corresponds to typical values in major arteries (Ref. 9). The flow dynamics analyzed in the rigid vessel of the ventricle chamber is seen to contribute to the instantaneous systolic and diastolic left ventricle model. Fig. 3 shows sampling of an elastic body experiencing deformation under the influence of moving fluid.
ventricle chamber. The separation phenomenon corresponds to the roll-up of the boundary layer and the formation of the vortex structure eventually travelling downstream (Ref. 10).

4. Simulation Results

Results of the 2D left ventricle simulation are laid out in graphical representation in Fig. 4. Using eight different plots, the velocity profile in the systolic and diastolic phase in the one second simulation time is observed with the timestep separated at approximately 0.1sec per plot. Fig. 4(a) and Fig. 4(b) shows the left ventricle simulation before initialization and at the first point of initialization. At this beginning, a positive separation of vorticity indicates a persistent negative shear forces in the portion of wall downstream of the enlargement. The same flow fields are reported in Fig. 4(c), Fig. 4(d) and Fig. 4(e). It can be observed that during systole the ventricle chamber is inflated with the incoming inflow from the mitral valve, pressuring the chamber to dilate and with that the flow is seen to accelerate at a maximum velocity profile throughout the chamber.

Fig. 4(f), Fig. 4(g) and Fig. 4(h) shows the blood flow profile during diastole. At diastole the boundary layer is thicker because of deceleration and the streamlines show about a zero net flow. The deceleration decreases the flow rate indicating vessel shrinks, not seen in the simulation but is assumed to increase the actual discharge and bulk velocity from the ventricle chamber out into the aortic. During the acceleration, the synchronous deformation of the wall leads to a less extended separated region, while the contraction of the vessel during the deceleration phase of the imposed diastolic pulse tends to diminish the value of the negative tangential stress.

5. Future Enhancement

The results presented in this thesis support the idea of computational models as a tool for understanding the blood flow in the left ventricle of the heart. However, since the intra-ventricular flow pattern is in real time, a three dimensional, future works should eventually be performed with a 3D virtual model.

The narrowing of the valve can be better simulated by introducing one or several cross links in between the three aortic valve leaflets which would allow for better closure during the ventricle dilation.

6. Conclusion

The work discussed in this paper provides a basic study on the blood flow pattern in the LV. The ALE method employed to incorporate the interaction between the blood flow and the leaflet motion has shown that simplified 2D ALE model can, within limits, predicts physiologically correct flow patterns in the LV. The model is also able to simulate forces exerted by the fluid during the mitral inflow and the aortic outflow with plausible prediction of the new displacement for the leaflet positions.

7. Acknowledgement

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8. References